

Worcester County Mathematics League

Freshman/JV Meet 2

December 13, 2017

Coaches' Copy
Rounds, Answers, and Solutions

WORCESTER COUNTY MATHEMATICS LEAGUE



Freshman Meet 2 – December 13, 2017

ANSWER KEY

Round 1: Algebraic Word Problems

1. 25 (South)
2. 34 (Douglas)
3. 668 (Assabet Valley)

Round 2: Number Theory

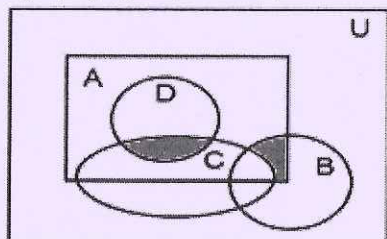
1. 18 (Shepherd Hill)
2. 0 and 6 (Quaboag)
3. 3432 (Assabet Valley)

Round 3: Operations on Numerical Fractions, Decimals, Percents, and Percentage Word Problems

1. 10.80 (QSC)
2. $\frac{169}{3600}$ (Westboro)
3. 5 (Hopedale)

Round 4: Set Theory

1. 2, 4, 5, 6, 7 (Auburn)



2. (Worcester)
3. 3 (Auburn)

Team Round

1. 64 (St. John's)
2. 14 (Notre Dame)
3. 5 (Westborough)
4. 30 (Nashoba)
5. 18 (Assabet Valley)
6. 21 (Hopedale)
7. 4,200 (Doherty)
8. 21 (QSC)

WORCESTER COUNTY MATHEMATICS LEAGUE



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Round 1: Algebraic Word Problems

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. At a bank, Dave exchanged a \$10 bill for an equal number of nickels, dimes, and quarters. How many nickels did Dave receive?
2. On July 13 in 1995, Mark's age was 3 times Jack's age. If Jack was a decade old on July 13, 1992, how old was Mark on July 13, 1990?
3. The Dakota Doll Company's cost of making a number of dolls is defined by the equation: $C = 35x + 100$, where C is the cost and x is the number of dolls. If each doll sells for \$50, how many dolls must be made and sold to realize a profit of \$9,920?

ANSWERS

(1 pt.) 1. _____ nickels

(2 pt.) 2. _____ years old

(3 pt.) 3. _____ dolls

WORCESTER COUNTY MATHEMATICS LEAGUE



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Round 2: Number Theory

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find the greatest common factor of 108, 126, and 180.

2. If the 9 digit number $8p2,5p7,p32$ is divisible by 2, 3, 6, 8, and 9, find all solutions for p .

3. Add $2012_3 + 2130_4 + 4342_5$ and give the answer in base 6. (X_b means that the number X should be understood to be in base b)

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. $p =$ _____

(3 pt.) 3. _____ (base 6)

WORCESTER COUNTY MATHEMATICS LEAGUE



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**Round 3: Operations on Numerical Fractions, Decimals, Percents, and
Percentage Word Problems**

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Haley went to the fabric store to buy fleece, during no tax weekend. The original price of one yard of fleece was \$15, but it was 70% off that day. At the register, her father paid 20% off the total, and all the tax. If Haley bought three yards of fleece, how much money did she spend?

2. Simplify to a single fraction: $\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}\right)^2$

3. Evaluate and express as a simplified fraction:

$$\left(\frac{\frac{3}{2} * 8}{\frac{2}{3} \div \frac{5}{18}}\right) * \left(3.25 - \left(\frac{5}{6} + 1\frac{5}{12}\right)\right)$$

ANSWERS

(1 pt.) 1. _____ dollars

(2 pt.) 2. _____

(3 pt.) 3. _____



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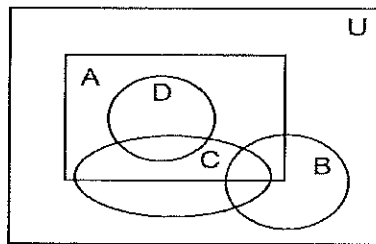
Round 4: Set Theory

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Set $A = \{1, 2, 3, 4, 5, 6\}$. Set $B = \{4, 5, 7\}$. Set $C = \{2, 4, 6, 8\}$. Find $(A \cup B) \cap (B \cup C)$.

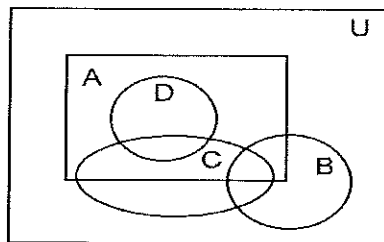
2. If A^c denotes the complement of set A and U is the universal set, in the diagram below, shade $((D \cap C) \cup (B \cap C^c)) \cap A$.



3. 30 students are on Corncob Academy's math team. Of those students, 16 take algebra, 10 take geometry, 15 take computer science, 7 take algebra and computer science, 3 take algebra and geometry, 6 take geometry and computer science, and 2 take algebra and geometry and computer science. How many students on the math team do not take any of these classes?

ANSWERS

(1 pt.) 1. _____



(2 pt.) 2.

(3 pt.) 3. _____ students

WORCESTER COUNTY MATHEMATICS LEAGUE



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Team Round

All answers must either be in simplest exact form or rounded to EXACTLY three decimal places, unless stated otherwise. (3 points each)

APPROVED CALCULATORS ALLOWED

1. What is the volume of a cube if its total surface area is 96 sq. centimeters?
2. Janice will need 64 meters of fence to enclose her rectangular yard for her dog. If the length of the yard is 4 meters longer than the width, what is the width of the yard?
3. Find n for the following equation

$$\frac{1}{2} \left(\frac{2}{3}\right) \left(\frac{3}{4}\right) \left(\frac{4}{5}\right) \left(\frac{5}{6}\right) \left(\frac{6}{7}\right) \left(\frac{7}{8}\right) \left(\frac{8}{9}\right) \left(\frac{9}{10}\right) \left(\frac{10}{11}\right) \left(\frac{11}{12}\right) = \frac{12}{2n^3 - 6}$$

4. The sum of three numbers is 98. The ratio of the first to the second is 2:3 and the ratio of the second to the third is 5:8. What is the second number?
5. How many seconds will it take to drive 792 feet at a speed of 30 mph?
6. There are 93 students being observed. In this group, 62 have brown hair, 49 have brown eyes, and 39 have both brown hair and brown eyes. How many students have neither brown hair nor brown eyes?
7. A total of \$9,000 is invested in two companies: The Misty Company, which returns 6% simple interest annually, and Liko Incorporated, which returns 3% simple interest annually. If the annual return from both investments is \$396, how much money is invested at The Misty Company?
8. Find the greatest common factor of 12,621 and 84,714.



Freshman Meet 2 – December 13, 2017 - SOLUTIONS

Round 1: Algebraic Word Problems

1. At a bank, Dave exchanged a \$10 bill for an equal number of nickels, dimes, and quarters. How many nickels did Dave receive?

Solution: First, we have to change every variable into the same unit, so we'll use 1,000 cents instead of 10 dollars. Let x be the number of coins, which means the price of a nickel is $5x$, a dime is $10x$, and a quarter is $25x$. We now have

$$5x + 10x + 25x = 1000$$

$$40x = 1000$$

$$x = 25$$

2. On July 13 in 1995, Mark's age was 3 times Jack's age. If Jack was a decade old on July 13, 1992, how old was Mark on July 13, 1990?

Solution: Let M be Mark's age in 1990 and J be Jack's age in 1990, which means $M + 5 = 3(J + 5)$ and $J + 2 = 10$. We can then calculate that $J = 8$. Using this, we can solve for the initial equation.

$$M + 5 = 3(8 + 5)$$

$$M + 5 = 3(13)$$

$$M + 5 = 39$$

$$M = 34$$

3. The Dakota Doll Company's cost of making a number of dolls is defined by the equation: $C = 35x + 100$, where C is the cost and x is the number of dolls. If each doll sells for \$50, how many dolls must be made and sold to realize a profit of \$9,920?

Solution: We know $C = 35x + 100$. The amount of money earned per doll can be represented as $50x$, which means the profit is equal to $50x - (35x + 100)$, or $15x - 100$.

$$15x - 100 = 9920$$

$$15x = 10020$$

$$x = 668$$



Freshman Meet 2 – December 13, 2017 - SOLUTIONS

Round 2: Number Theory

1. Find the greatest common factor of 108, 126, and 180.

Solution:

2		108	126	180
9		54	63	90
		6	7	10

$$\text{GCF} = 2 * 9 = 18$$

2. If the 9 digit number $8p2,5p7,p32$ is divisible by 2, 3, 6, 8, and 9, find all solutions for p .

Solution: We already know this unknown number is always divisible by 2 because it'll always be even for any value of p . Because a number that is divisible by 9 is always divisible by 3, and a number divisible by 9 and 2 is always divisible by 6, we only need to find the values for p that would make the 9 digit number divisible by 9 and 8.

To be divisible by 9, the sum of a number's digits must be divisible by 9. $8 + p + 2 + 5 + p + 7 + p + 3 + 2 = 27 + 3p$. 27 is 9×3 , so $3p$ equal a number divisible by 9. This means p can be equal to 0 ($0 \times 3 = 0$), 3 ($3 \times 3 = 9$), 6 ($6 \times 3 = 18$), and 9 ($9 \times 3 = 27$).

To be divisible by 8, the last three digits have to be divisible by 8. 32 and 632 are divisible by 8 while 332 and 932 are not. So p can be equal to 0 and 6.

3. Add $2012_3 + 2130_4 + 4342_5$ and give the answer in base 6. (X_b means that the number X should be understood to be in base b)

Solution: First, we have to change all numbers to base 10. $2012_3 = 2 + 3 + 0 + 54 = 59$, $2130_4 = 0 + 12 + 16 + 128 = 156$, and $4342_5 = 2 + 20 + 75 + 500 = 597$. The sum of these three numbers in base 10 is 812. To convert 812 in base 10 to base 6, we divide by 6 and use the remainder LCD to MSD. $812 / 6 = 135 \text{ R} = 2$, then $135 / 6 = 22 \text{ R} = 3$, then $22 / 6 = 3 \text{ R} = 4$, and finally $3 / 6 = 0 \text{ R} = 3$. Therefore, the answer is 3432_6 .



Freshman Meet 2 – December 13, 2017 - SOLUTIONS

Round 3: Operations on Numerical Fractions, Decimals, Percents, and Percentage Word Problems

1. Haley went to the fabric store to buy fleece, during no tax weekend. The original price of one yard of fleece was \$15, but it was 70% off that day. At the register, her father paid 20% off the total, and all the tax. If Haley bought three yards of fleece, how much money did she spend?

Solution: Three yards of fleece at original price would be equal to \$45. Since it is 70% off, Haley is only paying 30% of \$45, and $0.3 * 45 = 13.5$. The additional 20% off means she's paying 80% of \$13.50, and $0.8 * 13.5 = 10.8$, so Haley spent \$10.80.

2. Simplify to a single fraction: $(\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5})^2$

Solution: The fractions must be rewritten to have the same denominator, which will become

$$(\frac{30}{60} - \frac{20}{60} + \frac{15}{60} - \frac{12}{60})^2 = (\frac{13}{60})^2 = \frac{169}{3600}$$

3. Evaluate and express as a simplified fraction:

$$\left(\frac{\frac{3}{2} * 8}{\frac{2}{3} \div \frac{5}{18}}\right) * \left(3.25 - \left(\frac{5}{6} + 1\frac{5}{12}\right)\right)$$

Solution:

$$\begin{aligned} & \left(\frac{\frac{3}{2} * 8}{\frac{2}{3} \div \frac{5}{18}}\right) * \left(3.25 - \left(\frac{5}{6} + 1\frac{5}{12}\right)\right) \\ &= \left(\frac{\frac{3}{2} * 8}{\frac{2}{3} \div \frac{5}{18}}\right) * \left(\frac{13}{4} - \left(\frac{10}{12} + \frac{17}{12}\right)\right) \\ &= \left(\frac{3}{2} * 8 * \frac{3}{2} * \frac{5}{18}\right) * \left(\frac{13}{4} - \frac{27}{12}\right) \\ &= 5 * \left(\frac{39}{12} - \frac{27}{12}\right) = 5 * \frac{12}{12} = 5 \end{aligned}$$

Freshman Meet 2 – December 13, 2017 - SOLUTIONS

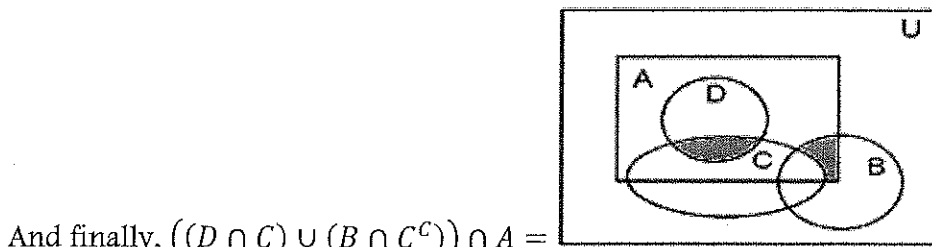
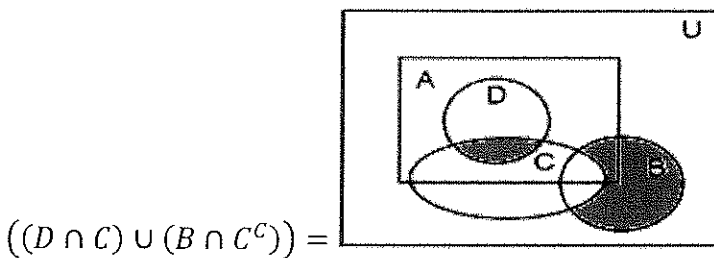
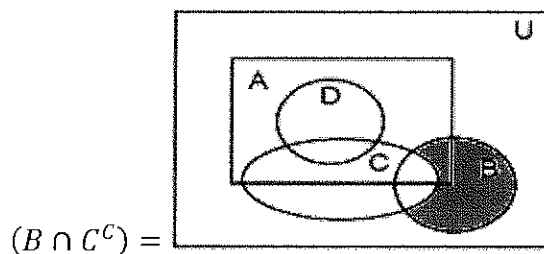
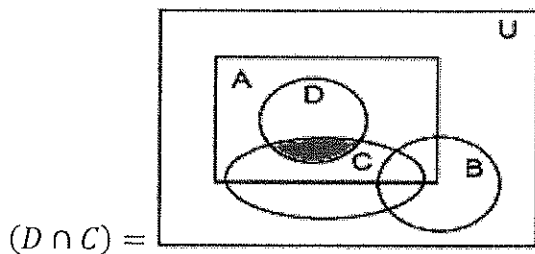
Round 4: Set Theory

1. Set $A = \{1, 2, 3, 4, 5, 6\}$. Set $B = \{4, 5, 7\}$. Set $C = \{2, 4, 6, 8\}$. Find $(A \cup B) \cap (B \cup C)$.

Solution: We have that $(A \cup B) = \{1, 2, 3, 4, 5, 6, 7\}$ and $(B \cup C) = \{2, 4, 5, 6, 7, 8\}$, so $(A \cup B) \cap (B \cup C) = \{2, 4, 5, 6, 7\}$.

2. If A^c denotes the complement of set A and U is the universal set, in the diagram below, shade $((D \cap C) \cup (B \cap C^c)) \cap A$.

Solution: We have that



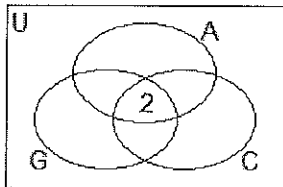


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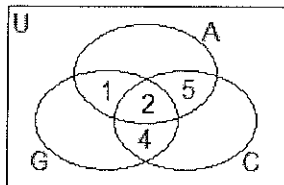
Round 4: Set Theory

3. 30 students are on Corncob Academy's math team. Of those students, 16 take algebra, 10 take geometry, 15 take computer science, 7 take algebra and computer science, 3 take algebra and geometry, 6 take geometry and computer science, and 2 take algebra and geometry and computer science. How many students on the math team do not take any of these classes?

Solution: We'll start with making a triple venn diagram and filling out the 2 students who take algebra (A), geometry (G), and computer science (C).



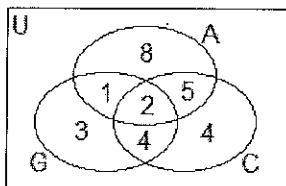
This means that the number of students who take only algebra and computer science is $7 - 2$, or 5. The number who take only algebra and geometry is $3 - 2$, or 1. The number who take only geometry and computer science is $6 - 2$, or 4. We can now fill in this information.



The number of students who take only algebra is $16 - (1 + 5 + 2) = 8$.

The number of students who take only geometry is $10 - (1 + 4 + 2) = 3$.

The number of students who take only computer science is $15 - (5 + 4 + 2) = 4$.



To find the number of students who take none of the classes, we can solve using the equation

$$30 - (8 + 3 + 4 + 1 + 5 + 4 + 2) = 30 - 27 = 3$$



Freshman Meet 2 – December 13, 2017 - SOLUTIONS

Team Round

1. What is the volume of a cube if its total surface area is 96 sq. centimeters?

Solution: The surface area of a cube can be defined as $6x^2$, where x is the length of one side.

$$6x^2 = 96$$

$$x^2 = 16$$

$$x = 4$$

The volume of a cube is defined as x^3 , and $4^3 = 64$ cubic centimeters.

2. Janice will need 64 meters of fence to enclose her rectangular yard for her dog. If the length of the yard is 4 meters longer than the width, what is the width of the yard?

Solution: Let w be the width of Janice's yard, so $w + 4$ is the length of her yard. The perimeter is equal to $2(l + w)$.

$$2(l + w) = 64$$

$$2((w + 4) + w) = 64$$

$$2w + 4 = 32$$

$$2w = 28$$

$$w = 14$$



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Team Round

3. Find n for the following equation

$$\frac{1}{2} \left(\frac{2}{3}\right) \left(\frac{3}{4}\right) \left(\frac{4}{5}\right) \left(\frac{5}{6}\right) \left(\frac{6}{7}\right) \left(\frac{7}{8}\right) \left(\frac{8}{9}\right) \left(\frac{9}{10}\right) \left(\frac{10}{11}\right) \left(\frac{11}{12}\right) = \frac{12}{2n^3 - 106}$$

Solution:

$$\frac{1}{2} \left(\frac{2}{3}\right) \left(\frac{3}{4}\right) \left(\frac{4}{5}\right) \left(\frac{5}{6}\right) \left(\frac{6}{7}\right) \left(\frac{7}{8}\right) \left(\frac{8}{9}\right) \left(\frac{9}{10}\right) \left(\frac{10}{11}\right) \left(\frac{11}{12}\right) = \frac{12}{2n^3 - 106}$$

$$\frac{1}{12} = \frac{12}{2n^3 - 106}$$

$$2n^3 - 106 = 144$$

$$2n^3 = 150$$

$$n^3 = 75$$

$$n = 5$$

4. The sum of three numbers is 98. The ratio of the first to the second is 2:3, and the ratio of the second to the third is 5:8. What is the second number?

Solution: Let n be the second number. Then the first number is $\frac{2}{3}n$, and the third is $\frac{8}{5}n$, so

$$\frac{2}{3}n + n + \frac{8}{5}n = 98$$

$$\frac{10 + 15 + 24}{15}n = 98$$

$$\frac{49}{15}n = 98$$

$$n = 98 * \frac{15}{49}$$

$$n = 30$$



Freshman Meet 2 – December 13, 2017 - SOLUTIONS

Team Round

5. How many seconds will it take to drive 792 feet at a speed of 30 mph?

Solution: First, we must note that there are 5,280 feet in a mile and 3,600 seconds in an hour. Then, we can solve using the following equation.

$$\frac{792 \text{ ft}}{1} * \frac{1 \text{ mi}}{5280 \text{ ft}} * \frac{1 \text{ hour}}{30 \text{ mi}} * \frac{3600 \text{ sec}}{1 \text{ hour}} = 18 \text{ seconds}$$

6. There are 93 students being observed. In this group, 62 have brown hair, 49 have brown eyes, and 39 have both brown hair and brown eyes. How many students have neither brown hair nor brown eyes?

Solution: We know that 39 students have both brown hair and brown eyes. To solve how for many have only brown hair, we compute $62 - 39 = 23$. To solve for how many have only brown eyes, we compute $49 - 39 = 10$. This means that the number of students who have brown hair, brown eyes, or both can be written as $23 + 10 + 39 = 72$, which means that $93 - 72 = 21$ students have neither brown eyes nor brown hair.

7. A total of \$9,000 is invested in two companies: The Misty Company, which returns 6% simple interest annually, and Liko Incorporated, which returns 3% simple interest annually. If the annual return from both investments is \$396, how much money is invested at The Misty Company?

Solution: Let x be the amount of money invested in The Misty Company. The information provided can be represented as

$$0.06x + 0.03(9000 - x) = 396$$

$$0.06x + 270 - 0.03x = 396$$

$$0.03x = 126$$

$$x = 4200$$

WORCESTER COUNTY MATHEMATICS LEAGUE



Freshman Meet 2 – December 13, 2017 - SOLUTIONS

Team Round

8. Find the greatest common factor of 12,621 and 84,714.

Solution: The prime factorization of 12,621 is $3 * 7 * 601$ (601 is prime by checking divisibility up through 23 since $24 < \sqrt{601} < 25$) and the prime factorization of 84,714 is $2 * 3 * 7 * 2017$ (2017 is prime by checking divisibility up through 43 since $44 < \sqrt{2017} < 45$). Therefore, the GCF of 12,621 and 84,714 is $3 * 7 = 21$.